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Lambda Calculus
Winter Term 2021/22
Exercise Sheet 5

Exercise 1 (Confluence & Commutation)

Show: If $\to_1$ and $\to_2$ are confluent, and if $\to_1^*$ and $\to_2^*$ commute, then $\to_{12} := \to_1 \cup \to_2$ is also confluent.

Exercise 2 (Strong Confluence)

A relation $\to$ is said to be *strongly confluent* iff:

$$t_2 \leftarrow s \to t_1 \Longrightarrow \exists u.\ t_2 \to^= u \ {}^*\!\!\leftarrow t_1$$

Show that every *strongly confluent* relation is also *confluent*.

Exercise 3 (Normal Forms)

Recall the inductive definition of the set NF of *normal forms*:

$$\frac{t \in \mathrm{NF}}{\lambda x.\ t \in \mathrm{NF}}$$

$$\frac{n \geq 0 \qquad t_1 \in \mathrm{NF} \qquad t_2 \in \mathrm{NF} \qquad \ldots \qquad t_n \in \mathrm{NF}}{x\ t_1\ t_2\ \ldots\ t_n \in \mathrm{NF}}$$

Show that this set precisely captures all normal forms, i.e.:

$$t \in \mathrm{NF} \Leftrightarrow \nexists t'.\ t \to_\beta t'$$

Homework 4 (Semi-Confluence)

A relation $\to$ is said to be *semi-confluent* iff:

$$t_2 \; {}^*\!\!\leftarrow s \to t_1 \Longrightarrow \exists u.\; t_2 \to^* u \; {}^*\!\!\leftarrow t_1$$

Show that $\to$ is *semi-confluent* if and only if it is *confluent*.

Homework 5 (Weak Diamond Property)

Assume that $\to$ has the following weaker diamond property:

$$t_2 \leftarrow s \to t_1 \wedge t_1 \neq t_2 \Longrightarrow \exists u.\; t_2 \to u \leftarrow t_1.$$

a) Is it still the case that every element is either in normal form or has no normal form?

b) Show that if t has a normal form, then all its reductions to its normal form have the same length.

Homework 6 (Normal Forms & Big Step)

Show:

$$t \in \mathrm{NF} \wedge t \Rightarrow_n u \Longrightarrow u = t$$