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Equational Logic
Summer Term 2015
Exercise Sheet 8

Exercise 39 (Critical Pairs)

Specify all critical pairs of the following term rewriting systems:

a) $f(g(f(x))) \longrightarrow x,\ f(g(x)) \longrightarrow g(f(x))$

b) $g(f(x)) \longrightarrow f(g^2(h(x))),\ h(f(x)) \longrightarrow f(g(h^2(x))),\ h(g(x)) \longrightarrow g(h(x))$

Which systems are locally confluent, which are convergent (i.e., terminating and confluent)?

Exercise 40 (Confluence)

Determine terms r_1 and r_2 such that $\{f(g(x)) \longrightarrow r_1,\ g(h(x)) \longrightarrow r_2\}$ is confluent.

Exercise 41 (Newman's Lemma)

Give an indirect proof of Newman's Lemma, by showing that $\longrightarrow$ has an infinite reduction sequence, if $\longrightarrow$ is locally confluent but not confluent.

Hint: Show that every element with two distinct normal forms has a direct successor with two distinct normal forms.

Homework 42 (Diamand Property)

Let $\longrightarrow$ be a TRS with the following "diamond property":

$$y \longleftarrow x \longrightarrow z \ \wedge\ y \neq z \quad \Longrightarrow \quad \exists u.\ y \longrightarrow u \longleftarrow z$$

Show that, if a has a normal form, all reductions from a to this normal form have the same length.

Homework 43 (Critical Pairs')

Specify all critical pairs of the following term rewriting systems:

a) $f(x,x) \longrightarrow a,\ f(x,g(x)) \longrightarrow b$

b) $f(f(x,y),z) \longrightarrow f(x,f(y,z)),\ f(x,a) \longrightarrow x$

c) $f(f(x,y),z) \longrightarrow f(x,f(y,z)),\ f(a,x) \longrightarrow x$

d) $0+y \longrightarrow y,\ x+0 \longrightarrow x,\ s(x)+y \longrightarrow s(x+y),\ x+s(y) \longrightarrow s(x+y)$

Which systems are locally confluent, which are convergent (i.e., terminating and confluent)?

Homework 44 (Completion)

Complete

$$E = \{f(g(f(x))) \approx x\}$$

to a convergent term rewriting system.