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SS 2016 | Exercise Sheet 2 | 20.04.2016

Submission of Homework: Before tutorial on Apr 27

Exercise 2.1. **[Predicate Logic]**
a) Specify a satisfiable formula F, such that for all models $\mathcal{A}$ of F, we have $|U_\mathcal{A}| \geq 3$.
b) Can you also specify a satisfiable formula F, such that for all models $\mathcal{A}$ of F, we have $|U_\mathcal{A}| \leq 3$?

Exercise 2.2. **[Resolution Completeness]**
a) Does $F \models C$ imply $F \vdash_{Res} C$? Proof or counterexample!
b) Can you prove $F \models C$ by resolution?

Exercise 2.3. **[Resolution of Horn-Clauses]**
Can the resolvent of two Horn-clauses be a non-Horn clause?

Exercise 2.4. **[Optimizing Resolution]**
We call a clause C *trivially true* if $A_i \in C$ and $\neg A_i \in C$ for some atom A_i. Show that the resolution algorithm remains complete if it does not consider trivially true clauses for resolution.

Exercise 2.5. **[Finite Axiomatization]**
Let M_0 and M be sets of formulas. M_0 is called *axiom schema* for M, iff for all assignments $\mathcal{A}$: $\mathcal{A} \models M_0$ iff $\mathcal{A} \models M$.

A set M is called *finitely axiomatized* iff there is a finite axiom scheme for M.

a) Are all sets of formulas finitely axiomatized? Proof or counterexample? b) Let $M = (F_i)_{i\in\mathbb{N}}$ be a set of formulas, such that for all i: $F_{i+1} \models F_i$, and not $F_{i+1} \models F_i$. Is M finitely axiomatized?

Homework 2.1. [Definitonal CNF] (3 points)
Calculate the definitional CNF of the following formula:

$$(A_1 \vee (A_2 \wedge \neg A_3)) \vee A_4$$

Homework 2.2. [Definitional DNF] (5 points)
We call formulas F and F' *equivalid* if

$$\models F \text{ iff } \models F'$$

First show that

$$F[G/A] \text{ and } (A \leftrightarrow G) \rightarrow F \text{ are equivalid}$$

for any formulas F and G and any atom F, provided that A does not occur in G. Now argue that for every formula F of size n there is an equivalid DNF formula G of size $O(n)$.

Homework 2.3. [Compactness Theorem] (5 points)
Suppose every subset of S is satisfiable. Show that then

every subset of $S \cup \{F\}$ is satisfiable or
every subset of $S \cup \{\neg F\}$ is satisfiable

for any formula F.

Homework 2.4. [Compactness and Validity] (2 points)
We say that a set of formulas S is valid if every F in S is valid. Prove or disprove:

$$S \text{ is valid iff every finite subset of } S \text{ is valid}$$

Homework 2.5. [Resolution] (5 points)
Use the resolution procedure to decide if the following formulas are satisfiable. Show your work (by giving the corresponding DAG or linear derivation)!

1. $(\neg A_1 \wedge A_2) \wedge (\neg A_1 \vee A_3) \wedge (A_1 \vee \neg A_2 \vee A_3)$
2. $A_2 \wedge (\neg A_3 \vee A_1) \wedge (\neg A_1 \vee A_2) \wedge (\neg A_1) \wedge (\neg A_2 \vee A_3)$