Logics Exercise

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SS 2018 — Exercise Sheet 7 — 23.05.2018

Submission of homework: Before tutorial on 29.05.2018. Until further notice, homework has to be submitted in groups of two students.

Exercise 7.1. [Herbrand Models]
Given the formula

$$F = \forall x \forall y (P(f(x), g(y)) \land \neg P(g(x), f(y)))$$

1. Specify a Herbrand model for F.
2. Specify a Herbrand structure suitable for F, which is not a model of F.

Exercise 7.2. [(In)finite Models]

1. Show that any model (for a formula of predicate logic) with a universe of size n can be extended to a model of size m for any $m \geq n$. Can it also be extended to an *infinite* model?
2. Now consider the extension of predicate logic with equality. Does above property still hold?

Exercise 7.3. [Natural Numbers and FOL]
We consider the following axioms in an attempt to model the natural numbers in predicate logic:

1. $F_1 = \forall x \forall y (f(x) = f(y) \rightarrow x = y)$
2. $F_2 = \forall x (f(x) \neq 0)$
3. $F_3 = \forall x (x = 0 \lor \exists y (x = f(y)))$

Give a model with an *uncountable* universe for:

1. $\{F_1, F_2\}$
2. $\{F_1, F_2, F_3\}$

Hint: A set S is uncountable if there is no bijection between S and $\mathbb{N}$.

Homework 7.1. **[Invalid Herbrand Models]** (8 points)
Recall the fundamental theorem from the lecture: "Let F be a closed formula in Skolem form. Then F is satisfiable iff it has a Herbrand model".

Explain "what goes wrong" if the precondition is violated: when F is not closed or not in Skolem form. Describe both cases.

Homework 7.2. **[Proof of the Fundamental Theorem]** (6 points)
Recall the fundamental theorem: Let F be a closed formula in Skolem form. Then F is satisfiable iff it has a Herbrand model. Give the omitted proof for the base case (slide 6, $\mathcal{A}(G) = \mathcal{T}(G)$).

Homework 7.3. **[Herbrand Models]** (6 points)
Given the formula

$$F = \forall x(P(f(x)) \leftrightarrow \neg P(x))$$

1. Specify a Herbrand model for F.
2. Specify a Herbrand structure suitable for F, which is not a model of F.