Prof. Tobias Nipkow
Lars Hupel

SS 2018 | Exercise Sheet 8 | 29.05.2018

Submission of homework: Before tutorial on 05.06.2018. Until further notice, homework has to be submitted in groups of two students.

Exercise 8.1. **[Simultaneous substitution]**
Recall that $F[t_1/x_1, \ldots, t_n/x_n]$ is the *simultaneous* substitution of $x_1, \ldots, x_n$ by $t_1, \ldots, t_n$.

1. Can we always express $F[t_1/x_1, \ldots, t_n/x_n]$ as a series of one-variable substitutions?
2. Can we always summarize a series of one-variable substitutions to a single simultaneous substitution?

Exercise 8.2. **[Occurs check]**
What happens if one omits the occurs check in the unification algorithm? Find an example where a unification algorithm without occurs check diverges or returns the wrong result.

Exercise 8.3. **[Unifiable terms]**
Specify the most general unifiers for the following sets of terms, if one exists:

$$\begin{aligned}
L_1 &= \{f(x, y), f(h(a), x)\} \\
L_2 &= \{f(x, y), f(h(x), x)\} \\
L_3 &= \{f(x, b), f(h(y), z)\} \\
L_4 &= \{f(x, x), f(h(y), y)\}
\end{aligned}$$

Exercise 8.4. **[Formulas without negation]**
Prove that every predicate logic formula that only contains $\wedge, \vee, \forall, \exists, \longrightarrow$ and atomic formulas is satisfiable. Is such a formula also valid?

Homework 8.1. **[Most general unifier]** (8 points)
Consider the unification problem $x \stackrel{?}{=} f(y)$. Without running the unification algorithm, prove that

1. $\sigma_1 = \{x \mapsto f(y)\}$ is a most general unifier.
2. $\sigma_2 = \{x \mapsto f(z), y \mapsto z\}$ is unifier, but not a most general unifier.

Hint: Argue using the definition of "most general unifier". Two substitutions σ and σ' can be proved equal by showing that they are equal on all variables, i.e., for all x, $x\sigma = x\sigma'$. Similarly, they can be proved not equal by demonstrating for a particular x that $x\sigma \neq x\sigma'$.

Homework 8.2. **[Unification]** (2 points)
Use the algorithm presented in the lecture to compute a most general unifier for the following set of formulas: $\{P(g(x), f(a)), P(y, x), P(g(f(z)), f(z))\}$

Homework 8.3. **[Untangling simultaneous substitution]** (5 points)
Recall Exercise 8.1. Demonstrate how to "untangle" a simultaneous substitution that has been obtained by consolidating one-variable substitutions back into one-variable substitutions.

Homework 8.4. **[Ground resolution]** (5 points)
Execute ground resolution to show that the following formula is unsatisfiable:

$$\forall x \forall y((P(x) \land \neg Q(y, y)) \to Q(x, y)) \land \neg \exists x(P(x) \land \exists y(Q(y, y) \land Q(x, y))) \land \exists y(P(y))$$