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SS 2018 | Exercise Sheet 10 | 12.06.2018

**Submission of homework:** Before tutorial on 19.06.2018. Until further notice, homework has to be submitted in groups of two students.

**Exercise 10.1.** [$\exists^*\forall^*$ **with Equality**]
Show that unsatisfiability of formulas from the $\exists^*\forall^*$ fragment with equality is decidable.

*Hint:* Reduce it to the $\exists^*\forall^*$-fragment without equality.

**Exercise 10.2.** [$\exists^*\forall^2\exists^*$]
Show how to reduce deciding unsatisfiability of formulas from the $\exists^*\forall^2\exists^*$-fragment to deciding unsatisfiability of formulas from the $\forall^2\exists^*$-fragment.

**Exercise 10.3.** [**Finite Model Property**]
A set of formulas $\mathcal{F}$ is said to have the *finite model property* if for all $F \in \mathcal{F}$, the following two statements are equivalent:

1. $F$ is satisfiable.
2. $F$ has a finite model.

Give a decision procedure for satisfiability of any such set of formulas.

**Exercise 10.4.** [**Sequent Calculus**]
Prove the following formulas in sequent calculus:

1. $\neg\exists x P(x) \to \forall x \neg P(x)$
2. $(\forall x(P \vee Q(x))) \to (P \vee \forall x Q(x))$

**Exercise 10.5.** [**Miniscoping**]
In the lecture, we proved that deciding unsatisfiability of monadic FOL formulas can be reduced to deciding unsatisfiability of formulas from the $\exists^*\forall^*$ fragment by using miniscoping.

Prove the lemma that after miniscoping, no nested quantifiers remain.

**Homework 10.1. [FOL without Function Symbols]** (8 points)
Describe an algorithm that transforms any formula (in FOL with equality) into an equisatisfiable formula (in FOL with equality) that does not use function symbols.

*Hints:* Functions can be modelled as relations satisfying some additional properties. Don't forget to deal with constants, i.e., functions with arity 0. A similar transformation as in the previous exercise might be helpful.

**Homework 10.2. [Sequent Calculus]** (6 points)
Prove the following statements using sequent calculus if they are valid, or give a countermodel otherwise.

1. $\neg\forall x \exists y \forall z(\neg P(x,z) \wedge P(z,y))$
2. $\forall x \forall y \forall z(P(x,x) \wedge (P(x,y) \wedge P(y,z) \rightarrow P(x,z)))$

*Note:* While you are free to carry out the sequent calculus proofs in Logitext, application of $\forall L$ and $\exists R$ delete the principal formula. You have to select "Contract" first before instantiating the principal formula.

**Homework 10.3. [A Strange Island]** (6 points)
You are visiting an island. It is inhabited by two kinds of people: *knights* and *knaves.* Knights always tell the truth and knaves always lie.

You interview all inhabitants. Every inhabitant tells you "we are all of one kind".

1. Model this situation as a formula in first-order logic.
2. Give a model that corroborates the story, or alternatively explain why it is contradictory. Use any calculus from the lecture for that (e.g. resolution).