Logics Exercise

TU München
Institut für Informatik

Prof. Tobias Nipkow
Lars Hupel

SS 2018 | Exercise Sheet 13 | 03.07.2018

Submission of homework: Before tutorial on 10.07.2018. Until further notice, homework has to be submitted in groups of two students.

Exercise 13.1. **[Presburger Arithmetic]**
Eliminate the quantifiers from the following formulas, according to Presburger arithmetic:

1. $\forall y(3 < x + 2 \cdot y \vee 2 \cdot x + y < 3)$
2. $\forall x(\exists y(x = 2 \cdot y) \rightarrow (2 \cdot x \geq 0 \vee 3 \cdot x < 2))$

Exercise 13.2. **[Quantifier Elimination for $Th(\mathbb{N}, 0, S, =)$]**
Give a quantifier-elimination procedure for $Th(\mathbb{N}, 0, S, =)$ where S is the successor operation on natural numbers, i.e. $S(n) = n + 1$.

Hint: $a = b$ iff $S^k(a) = S^k(b)$ for any $a, b, k \in \mathbb{N}$.

Exercise 13.3. **[Brainstorming]**
Collect about 30 important pieces of terminology that were used in the lecture and that you want to remember for the exam.

Also collect the three results that you find most interesting.

Homework 13.1. **[Quantifier Elimination]** (8 points)
Perform Presburger arithmetic quantifier elimination for each formula:

1. $\forall x \forall y(0 < y \wedge x < y \rightarrow x + 1 < 2 \cdot y)$
2. $\forall x(\exists y(x = 2 \cdot y \wedge 2 \mid y) \rightarrow 4 \mid x)$

Homework 13.2. **[Quantifier Elimination for $Th(\mathbb{Z}, 0, S, P, =, <)$]** (8 points)
Give a quantifier-elimination procedure for $Th(\mathbb{Z}, 0, S, P, =, <)$ where S is the successor and P the predecessor operation on integers, i.e. $S(n) = n + 1$ and $P(n) = n - 1$. Do not use Presburger arithmetic; give a direct algorithm.