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Semantics of Programming Languages

Exercise Sheet 4

It's blatantly clear
You stupid machine, that what
I tell you is true

— Michael Norrish

Unlike the previous exercises, where the main challenge was to get the definitions right, we now look at situations where Isabelle needs some guidance to find the proof, even though the properties are intuitively clear.

Exercise 4.1 An alternative introduction rule for $\rightarrow*$

We consider the inductive definition of $\rightarrow*$ from theory `Inductive_Demo`. Prove the following lemma, first in apply-style, and then in a structured Isar proof.

lemma *steps_right*: "$[\![\ x \rightarrow* y;\ y \rightarrow z\ ]\!] \Longrightarrow x \rightarrow* z$"

Exercise 4.2 Palindromes

Formalize the following inductive definition as a predicate *palindrome* :: *nat list* $\Rightarrow$ *bool*:

- The empty list and a singleton list is a palindrome.
- If *xs* is a palindrome, then so is $a\#xs@[a]$.

Prove the following property:

lemma *palindrome_rev*: "*palindrome xs* $\longleftrightarrow$ (*rev xs* = *xs*)"

Homework 4

Submission until Wednesday, November 24, 2010, 12:00 (noon).

Context-free grammars (CFGs) can be modelled as inductive definitions, just like *palindrome* above. We consider the set of valid (i.e., balanced) sequences of parentheses.

The most natural definition of valid sequences of parentheses can be written as the following grammar over the alphabet $\{(,)\}$:

$$S \quad \rightarrow \quad \varepsilon \quad | \quad (\; S\;) \quad | \quad S\, S$$

where ε is the empty word.
A second, somewhat unusual grammar is the following one:

$$T \quad \rightarrow \quad \varepsilon \quad | \quad T\; (\; T\;)$$

(a) Model both grammars as inductive sets S and T. As the type of symbols, you can simply use the following datatype, where A and B stand for opening and closing parentheses:

datatype *symbol* $= A \mid B$

(b) Prove that $s = t$, first in apply-style, and then with a structured Isar proof.