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Semantics of Programming Languages

Exercise Sheet 1

Before beginning to solve the exercises, open a new theory file named `Ex01.thy` and write the the following three lines at the top of this file.

theory *Ex01*
imports *Main*
begin

Exercise 1.1 Calculating with natural numbers

Use the **value** command to turn Isabelle into a fancy calculator and evaluate the following natural number expressions:

"2 + (2::nat)" *"(2::nat) * (5 + 3)"* *"(3::nat) * 4 - 2 * (7 + 1)"*

Can you explain the last result?

Exercise 1.2 Natural number laws

Formulate and prove the well-known laws of commutativity and associativity for addition of natural numbers.

Exercise 1.3 Counting elements of a list

Define a function which counts the number of occurrences of a particular element in a list.

fun *count* :: *"'a list ⇒ 'a ⇒ nat"*

Test your definition of *count* on some examples and prove that the results are indeed correct.

Prove the following inequality (and additional lemmas, if necessary) about the relation between *count* and *length*, the function returning the length of a list.

theorem *"count xs x ≤ length xs"*

Exercise 1.4 Adding elements to the end of a list

Recall the definition of lists from the lecture. Define a function *snoc* that appends an element at the right end of a list. Do not use the existing append operator @ for lists.

fun *snoc* :: *"'a list ⇒ 'a ⇒ 'a list"*

Convince yourself on some test cases that your definition of *snoc* behaves as expected, for example run:

value *"snoc [] c"*

Also prove that your test cases are indeed correct, for instance show:

lemma *"snoc [] c = [c]"*

Next define a function *reverse* that reverses the order of elements in a list. (Do not use the existing function *rev* from the library.) Hint: Define the reverse of $x \# xs$ using the *snoc* function.

fun *reverse* :: *"'a list ⇒ 'a list"*

Demonstrate that your definition is correct by running some test cases, and proving that those test cases are correct. For example:

value *"reverse [a, b, c]"*
lemma *"reverse [a, b, c] = [c, b, a]"*

Prove the following theorem. Hint: You need to find an additional lemma relating *reverse* and *snoc* to prove it.

theorem *"reverse (reverse xs) = xs"*

Homework 1 The doubling function

Submission until Tuesday, October 23, 10:00 am.

This homework is to be done both with pen-and-paper and with Isabelle.

You will define recursively the function *double* that takes one number and returns its double. For example, we have $double(3) = 6$. Below, by "numbers" we mean "natural numbers".

Your first task is to define $double(n)$ recursively on n—that is to say, define $double(0)$ and then define $double(Suc(n))$ in terms of $double(n)$. You are not allowed to use addition or multiplication in the definition.

Another task is to prove that $double(m + n) = double\ m + double\ n$ for all numbers m and n.

Finally, you have to prove that your recursive definition of *double* is correct, in that $double\ n = n + n$ for all numbers n.