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Semantics of Programming Languages

Exercise Sheet 2

Homework 2.1 Tree traversal

Submission until Tuesday, November 8, 10:00am.

Recall the tree definition from the lecture and the function *mirror* to mirror trees:

datatype *'a tree = Tip | Node "'a tree" 'a "'a tree"*

fun *mirror* :: *"'a tree ⇒ 'a tree"* **where**
"mirror Tip = Tip" |
"mirror (Node l x r) = Node (mirror r) x (mirror l)"

Define a function *in_order*, which traverses a tree in in-order. Prove that your definition of *in_order* fulfills the specification

theorem
"rev (in_order t) = in_order (mirror t)"

where *rev* is the predefined function for reversing lists.

Homework 2.2 Tail-Recursive Form of Addition

Submission until Tuesday, November 8, 10:00am.

The list-reversing function *itrev* is an example of a *tail-recursive* function: Note that the right-hand side of the second equation for *itrev* is simply an application of *itrev* to different arguments.

fun *itrev* :: *"'a list ⇒ 'a list ⇒ 'a list"* **where**
"itrev [] ys = ys" |
"itrev (x#xs) ys = itrev xs (x#ys)"

In this homework problem you will define a tail-recursive version of addition for natural numbers, and prove that it is associative and commutative.

First, define a function *add* :: *nat ⇒ nat ⇒ nat* in Isabelle that calculates the sum of its arguments. Like *itrev*, your definition should be tail-recursive: That is, in the recursive case the right-hand side should only be an application of *add* to different arguments.

fun *add* :: *"nat ⇒ nat ⇒ nat"*

Next, you must prove that *add* is associative. Hint: The proof will require at least one additional lemma. Also remember that some proofs by induction may require generalization with *arbitrary*.

theorem *"add (add x y) z = add x (add y z)"*

Finally, you must prove that *add* is commutative. This may require more lemmas in addition to those used for the associativity proof.

theorem *"add x y = add y x"*