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Semantics of Programming Languages

Exercise Sheet 11

Exercise 11.1 Sign Analysis

Instantiate the abstract interpretation framework to a sign analysis over the lattice $pos, zero, neg, any$, where *pos* abstracts positive values, *zero* abstracts zero, *neg* abstracts negative values, and any abstracts any value.

datatype *sign* = *Pos* | *Zero* | *Neg* | *Any*

Exercise 11.2 AI for Conditionals

Our current constant analysis does not regard conditionals. For example, it cannot figure out, that after executing the program *x*:=*2*; *IF x<2 THEN x*:=*2 ELSE x*:=*1*, *x* will be constant.
In this exercise, we extend our abstract interpreter with a simple analysis of boolean expressions. To this end, modify locale *Val_semilattice* in theory *Abs_Int0.thy* as follows:

- Introduce an abstract domain *'bv* for boolean values, add, analogously to *num'* and *plus'* also functions for the boolean operations and for *less*.
- Modify *Abs_Int0* to accommodate for your changes.

Homework 11.1 Complete Lattices

Submission until Monday, January 27, 10:00am.

***Note:* This homework could be a typical exam exercises. It is not done in Isabelle, so either submit your answer in a text block in the theory file that you turn in or submit it on paper before the start of the next tutorial.**

Which of the following ordered sets are complete lattices?

- $\mathbb{N}$, the set of natural numbers $\{0, 1, 2, 3, \ldots\}$ with the usual order
- $\mathbb{N} \cup \{\infty\}$, the set of natural numbers plus infinity, with the usual order and $n < \infty$ for all $n \in \mathbb{N}$.
- A finite set A with a total order $\leq$ on it.
- $\mathbb{B}^*$, the set of all lists of booleans $\{[], [\texttt{True}], [\texttt{False}], \ldots\}$, with prefix order: $a \leq b \Longleftrightarrow \exists c.\ b = a@c$.
- The set of all lists of a certain length n if a point-wise ordering is used.

For each ordered set, prove (no Isabelle proof) that it is a complete lattice or give a counter example. Note: You do not need to prove the order properties, only the properties that constitute complete lattice.

Homework 11.2 Using the VCG

Submission until Monday, January 27, 10:00am.

Instantiate the abstract interpretation framework to find out which variables have values in the range $\{-128 \ldots 127\}$, i.e. fit into one byte.

Start your development from *HOL−IMP.Abs_Int1_parity*. You do not need to show termination.

Test your framework on the test cases for constant propagation from *HOL−IMP.Abs_Int_Tests*.